Geothermics (MSc. Oil-mining eng. specialization, 2020)

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Temperature Scales

These scales were developed by Fahrenheit(1714), Celsius (1742), Kelvin(1815), respectively.

Degrees Fahrenheit, (developed in 1714 by G. Daniel Fahrenheit) are mainly used by North American meteorologists. For meteorological records the rest of the world applies **degrees Celsius** (introduced by him in 1742). The conversion from units of **degrees Fahrenheit (^{0}F)** to **degrees Celsius (^{0}C)**: $$^0C = (^0F - 32)/1.8$$

The third temperature scale is by scientists, it is the **Kelvin** scale (1815),

which begins at **absolute zero** (resulting in no negative temperature values). Its divisions are the same as the Celsius scale. The equation to convert **degrees Celsius (^{0}C)** into **Kelvin (K)** is :

$$K = {}^0C + 273.16$$

Temperature Scales

Fahrenheit	Celsius	Kelvin	
212	100	373	Boiling point of water at sea-level
194	90	363	
176	80	353	
158	70	343	
140	60	333	
122	50	323	
104	40	313	
86	30	303	
68	20	293	Average room temperature
50	10	283	
32	0	273	Melting (freezing) point of ice (water) at sea-level
14	-10	263	
-4	-20	253	
-22	-30	243	
-40	-40	233	
-58	-50	223	
-76	-60	213	
-94	-70	203	
-112	-80	193	
-130	-90	183	-89°C (-129 °F) Lowest recorded temperature Vostok, Antarctica July, 1983
-148	-100	173	

Reference: Ahrens (1994)

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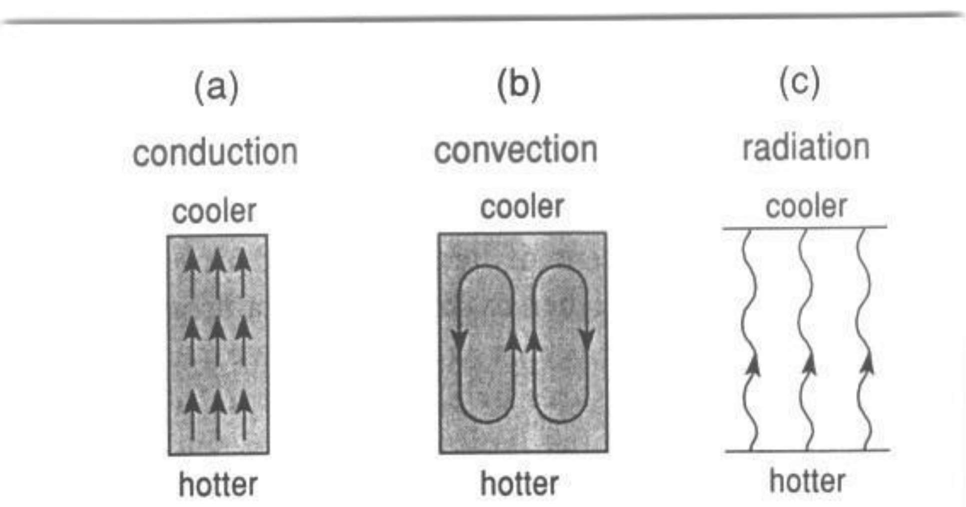
adapted from: Ahrens

Fourier equations (1822) can be applied for heat conduction (there are three types of heat transport: conduction, convection, radiation)

Kelvin (1863) observed that there was an increasing temperature variation with depth in borehole and due to this temperature-depth relationship he thought that heat was transported from greater depth upward, to the surface.



Ways that heat travels.

Radiation cannot be observed in geological situation.

Mussett&Khan, 2000

Heat conduction- Fourier' 1st law

In the crust it is the most common form of heat propagation. In this case the transfer of heat can be observed due purely to a temperature difference. It is a diffusive process, in which molecules and atoms transmit kinetic energy to their neighbouring particles. The Fourier's law of heat conduction states:

$$\vec{q} = -\lambda gradT = -\lambda\vec{G}$$

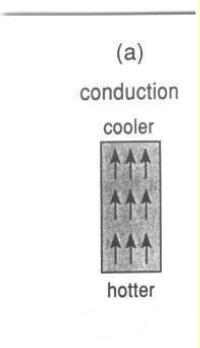
$$gradT = \frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}$$

where *q* is the heat flux vector, *λ* is the thermal conductivity (scalar or tensor), and grad*T* is the temperature gradient. The minus sign indicates that heat is transferred from higher temperature to lower temperature regions.

If we assume a heat conduction only in vertical direction, then

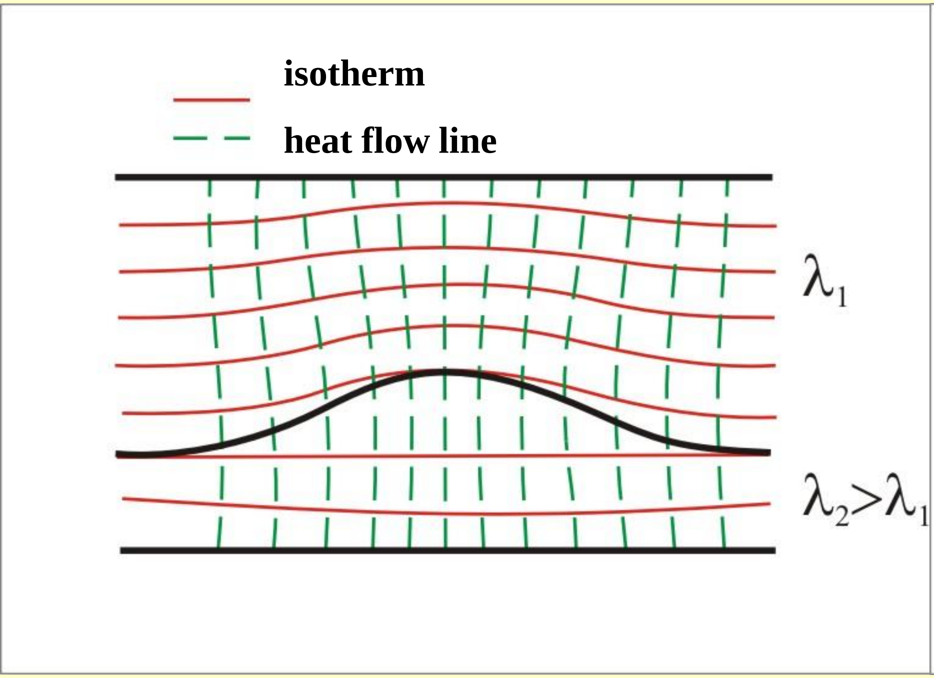
$$gradT = \frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k} = \frac{\partial T}{\partial z}\vec{k} = \vec{G} \approx \frac{\Delta T}{\Delta z}\vec{k}$$

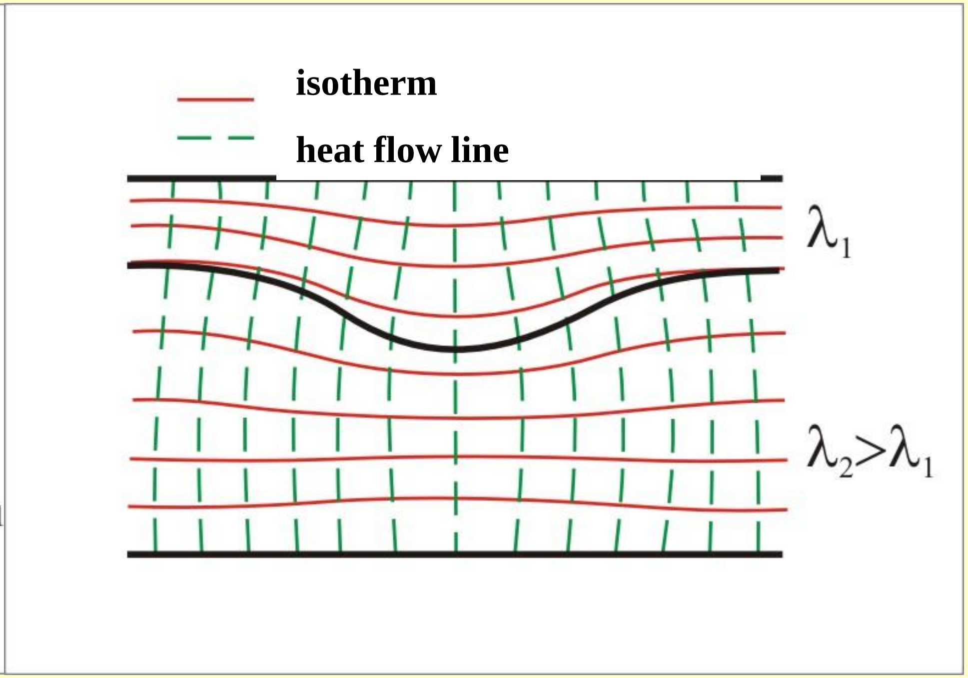
The unit of heat flux is W/m^2 (the flow of heat energy per unit area and per unit time). It is often called heat flow density or heat flow in geophysics. The unit of temperature gradient is $^0C/m$ or K/m. The unit of thermal conductivity is W/m^0C.



We can present the heat transfer visually by heat flow and isothermal lines.

The ***heat flow lines*** show the direction of heat flow and they are **perpendicular** to the ***isothermal lines*** (lines with constant temperature). The ***heat flux vector*** at an arbitrary point also orthogonal to the ***isotherm*** at that point.





Heat conduction- Fourier' 2nd law

Fourier' 2nd law defines the temperature distribution in the function of time and space:

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c}\Delta T = k\Delta T$$

If 1D heat transfer is assumed

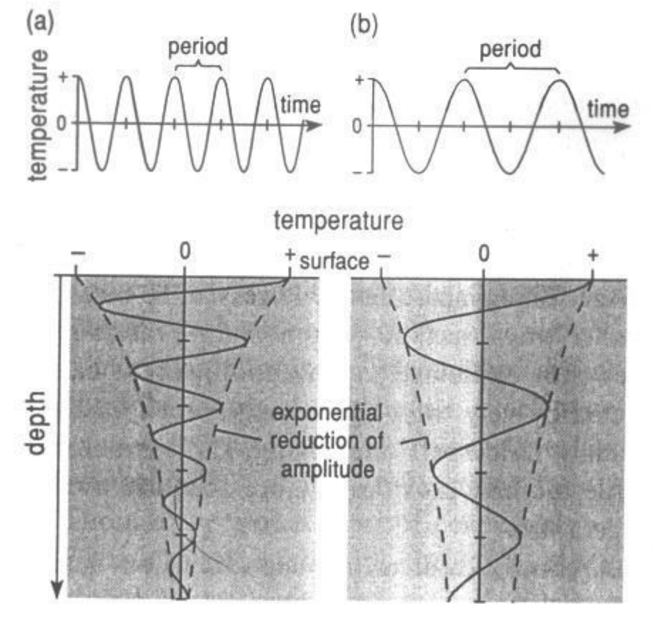
$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho c}\Delta T = k\frac{\partial^2 T}{\partial z^2}$$

The partial derivative of temperature with respect time is proportional to the second partial derivative of temperature with respect space co-ordinates.

Here ρ denotes density, c stands for specific heat capacity, its unit is J/kg ^{0}C. In this diffusion equation $k = \lambda / \rho c$ denotes thermal diffusivity with dimensions m^2s^{-1} .

This equation can be applied to investigation on the penetration of external heat into the ground etc.

Mussett&Khan, 2000



Propagation of alternating temperature changes at two frequencies.

Heat convection

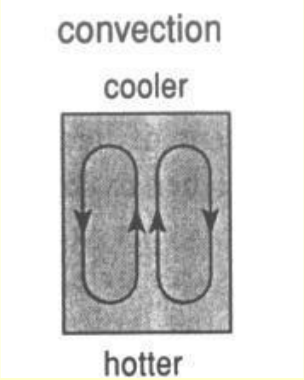
Transfer of heat by mass movement, by motion of the medium.

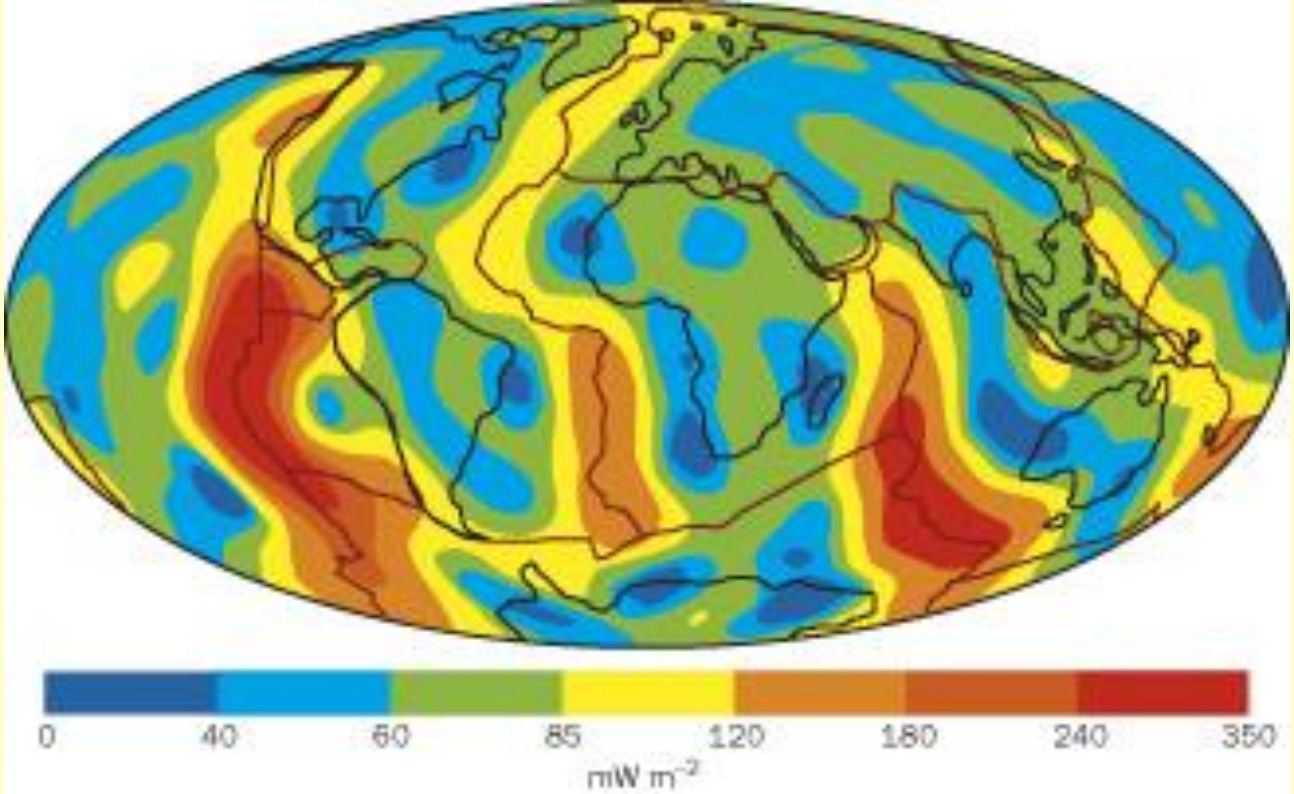
$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho\, c}\Delta T + \vec{v}\; gradT$$

Fourier-Kirchhoff equation, all parameters refer to the fluid transporting heat.

$$\rho c \frac{\partial T}{\partial t} = \lambda \Delta T + \rho^{f} c^{f} \vec{v}^{D} gradT + S$$

In porous, isotropic formation where the physical parameters without indices refer to the homogeneous rock, while f index refers to the fluid, D denotes Darcy seepage velocity and S stands for source term (more details Bobok, 1995)





Pollack et.al. 1993

Global heatflow map (in dimensions of mW/m^2)

It is based upon 24774 data gained at 20201 observation points. The contours show a degree and order 12 spherical harmonic representation. The regions with higher heat flux coincide with oceanic ridge system, the heat flow over average value is due to convection. There are low heat flow values over ancient part of continental plates (Canadian Shield, Scandinavian Shield, Angara Shield, African Shields, West Australia, etc.)

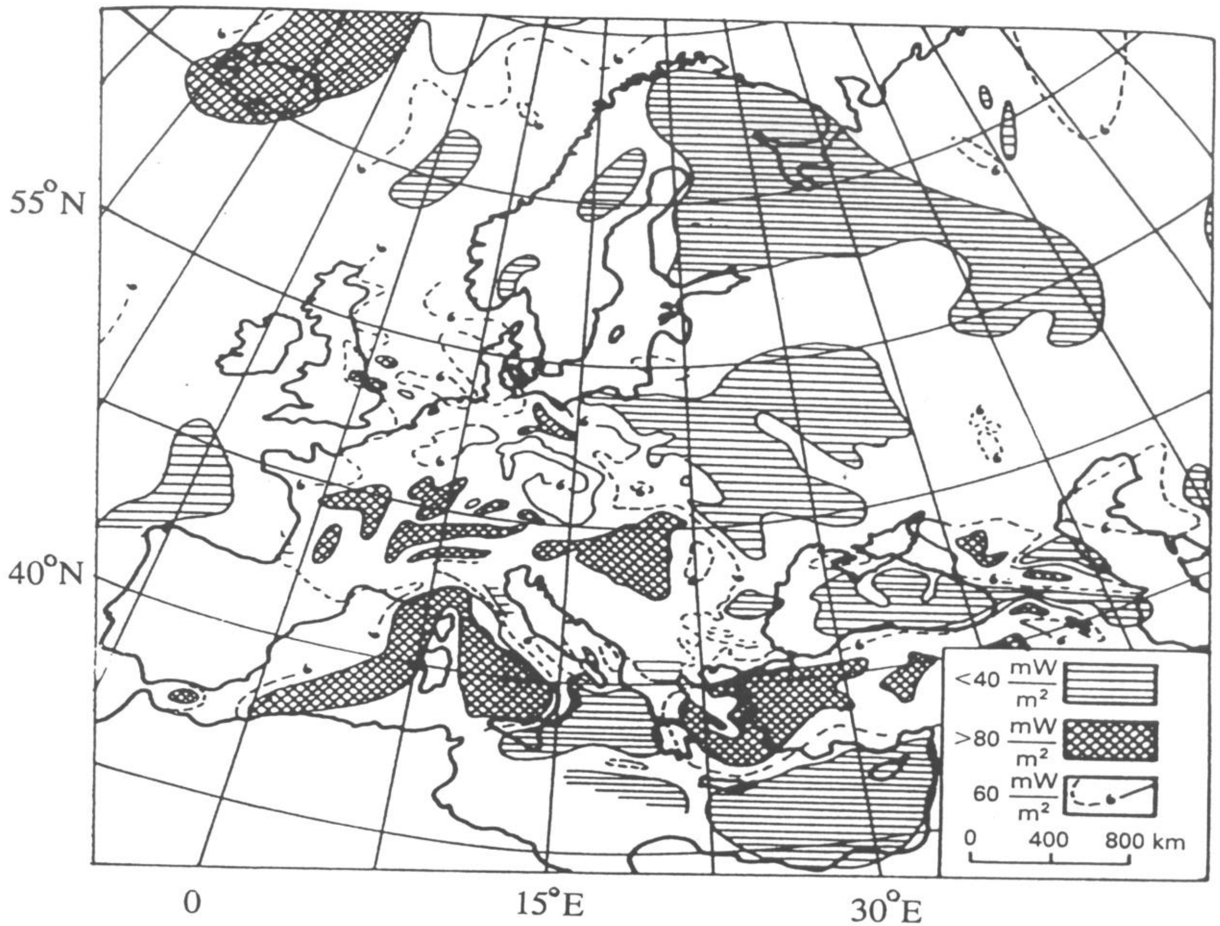
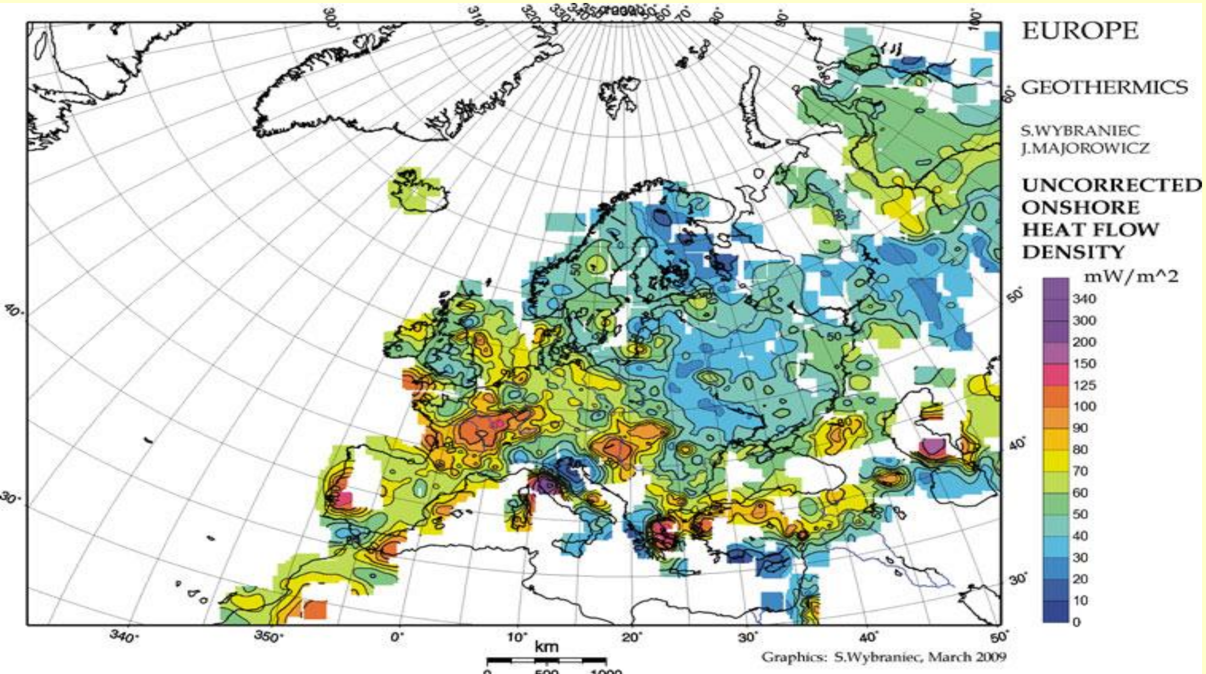
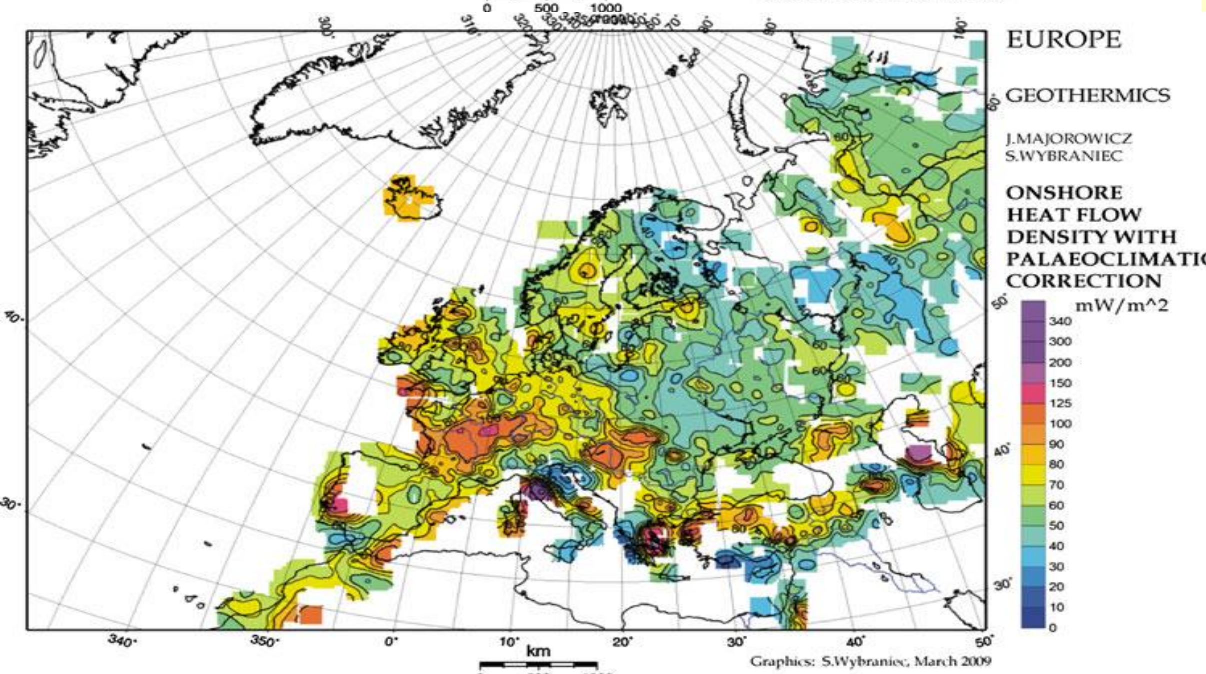


Fig. 10.2 Simplified version of heat-flow map of Europe (After Čermák, 1984.) The lowest heat-flow values are in the Baltic and Ukrainian shields, whereas the highest values are in the young mountainous regions of the Alps, Carpathians, and Caucasus.

Average heat flux in Baltic Shield and Ukrainian Shield is between 30-50 mW/m² , and it is greater than 60 mW/m² in the Alps, Carpathians and Caucasus. Apart from regions showing thermal changes at present, these heat flow values have been developed due to the superposition of earlier processes with thermal effects.





New heat flow map of Europe is based on updated database of uncorrected heat flow values to which paleoclimatic correction was applied for the continent. Correction is depth dependent . The most significant factor in the course of correction the glacial– interglacial history was due to its largest impact. It is obvious that large part of the uncorrected heat flow values in the existing heat flow databases from wells as shallow as few hundreds of meters was underestimated. This explains some very low uncorrected heat flow values 20–30 mW/m^2 in the shields and shallow basin areas of the craton.

Ref: [Majorowicz](), [Wybraniec]() : New terrestrial heat flow map of Europe after regional paleoclimatic correction application , Int. Journal of Earth Sciences, 2010

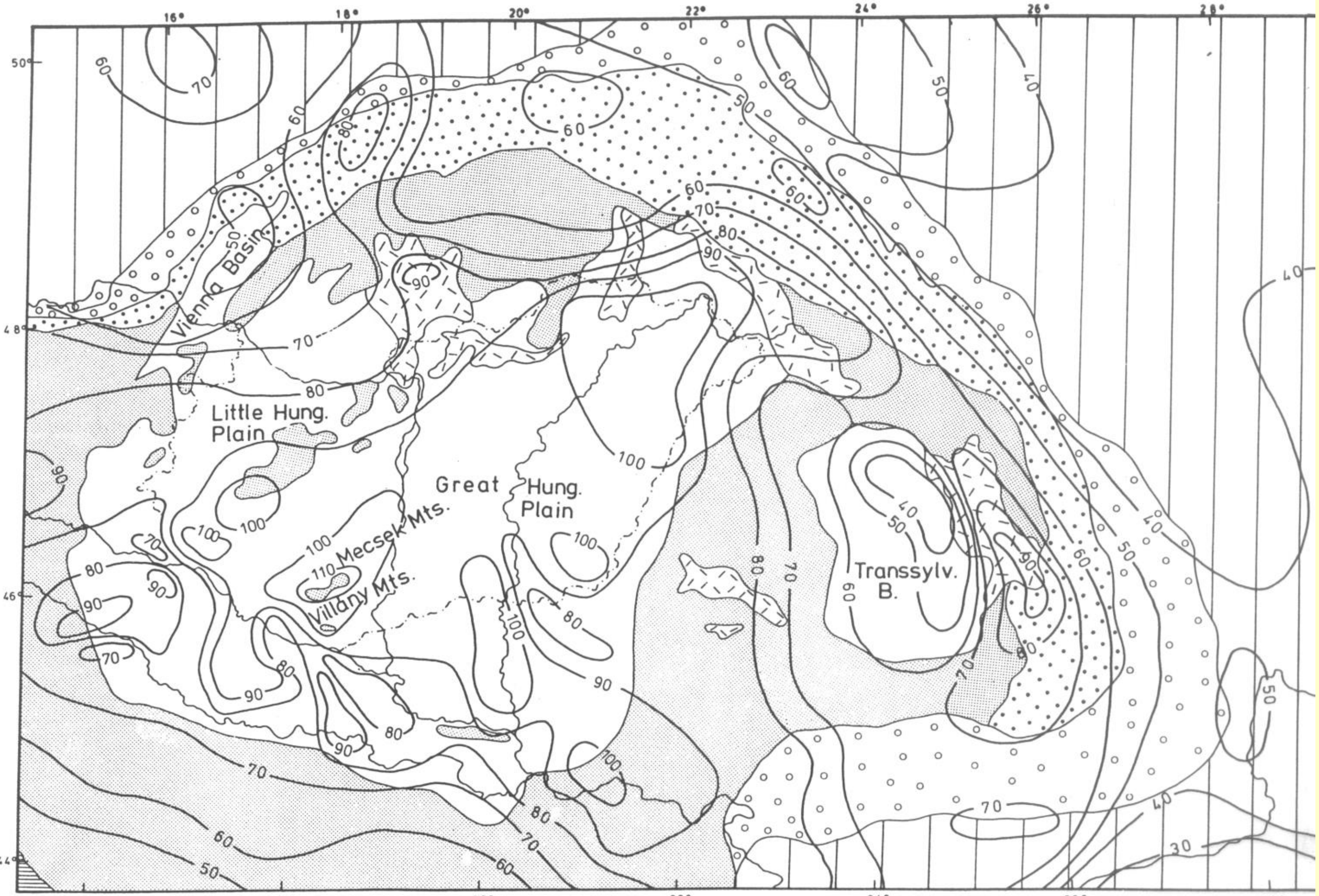
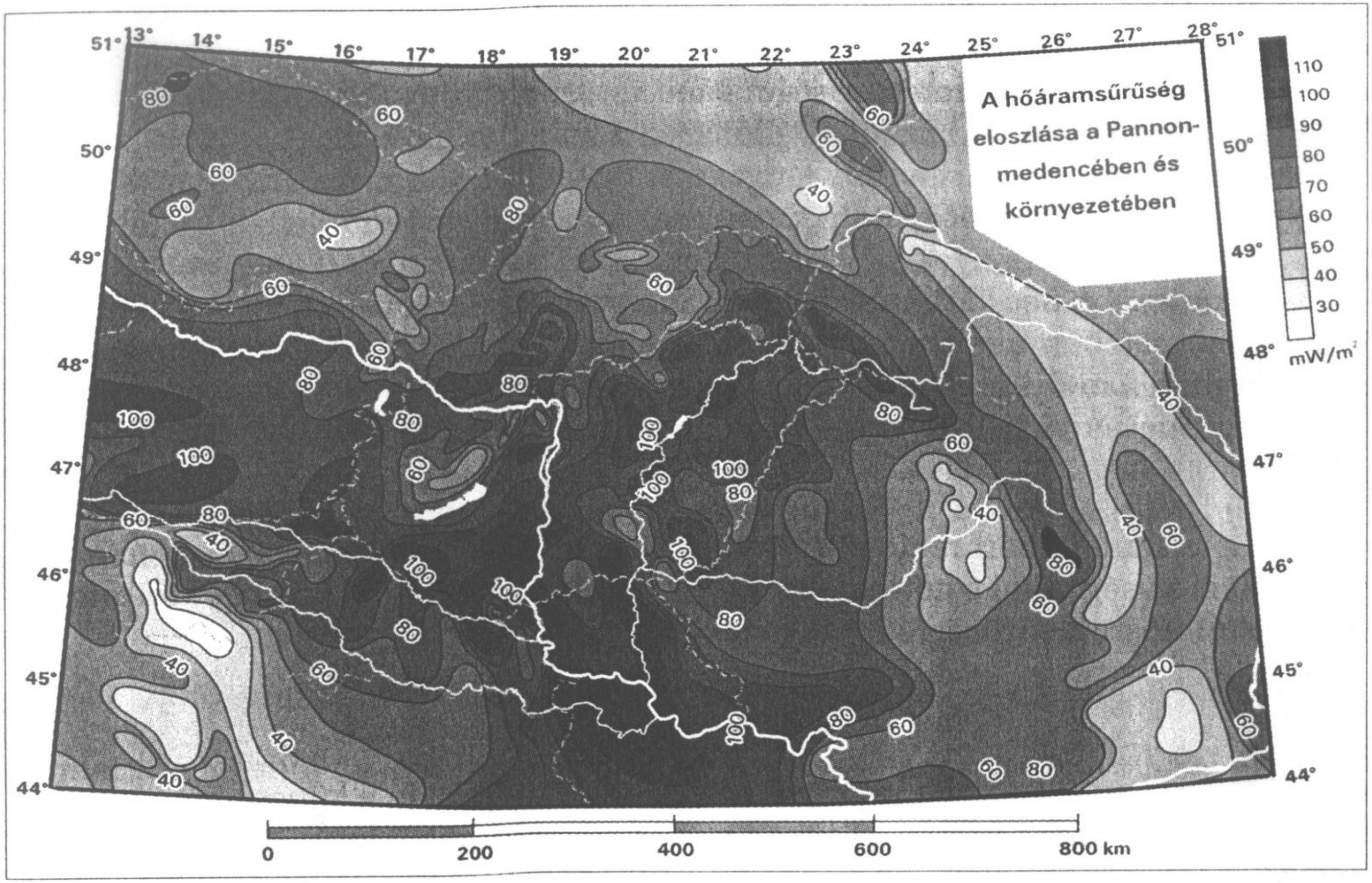


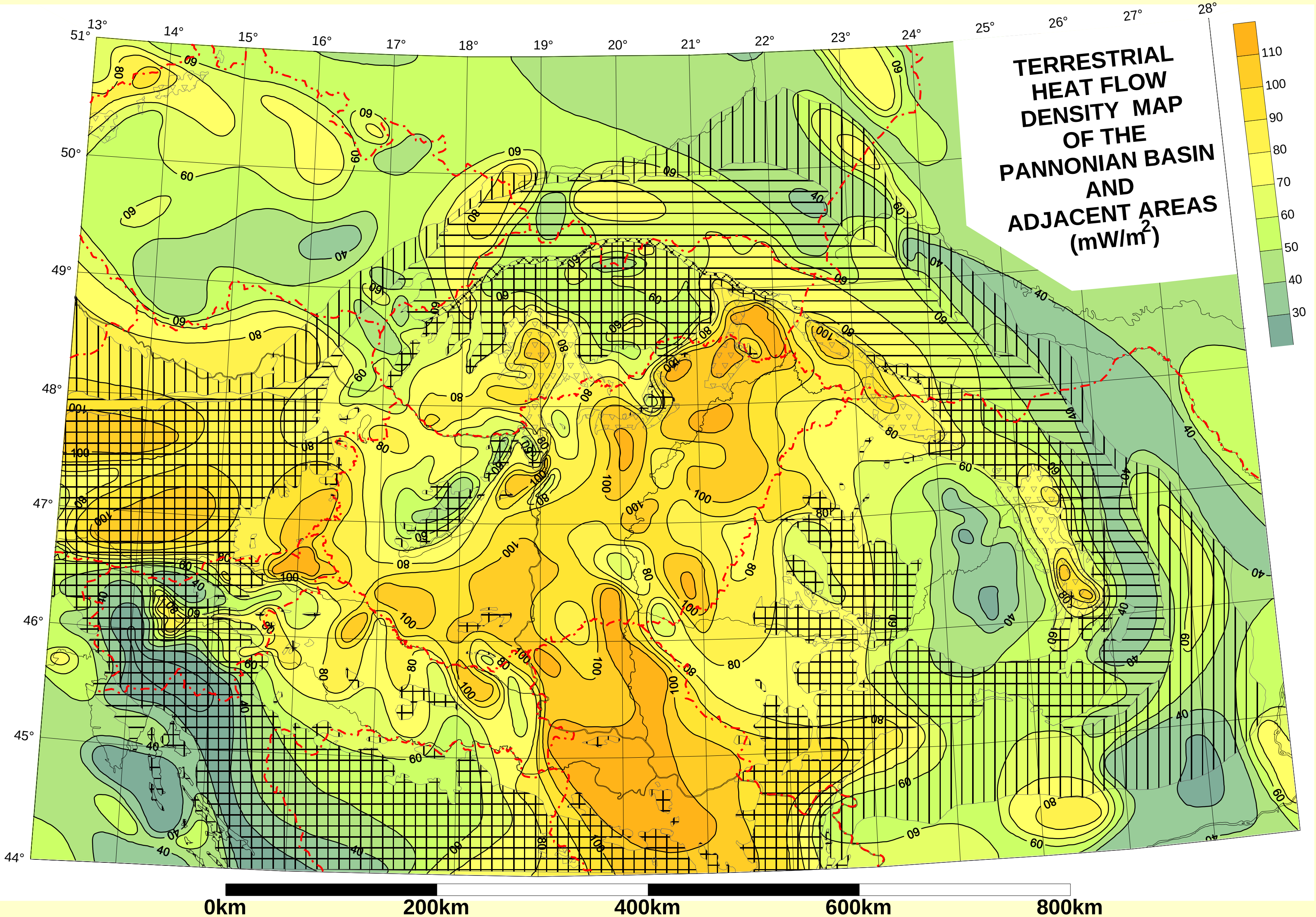
Figure 8.8. Isolines of heat flow in mW/m^2 units for the Carpatho-Pannonian area and its surroundings. Legend see in Figure 8.5 (modified Čermák and Hurtig, 1979)

The geothermal gradient in Pannonian Basin is 50-60 ^{0}C/km, the continental average is 30 ^{0}C/km. The heat flow (or heat flux) is 80-100mW/m^2, elsewhere 62mW/m^2. There are (must be) mainly low enthalpy geothermal reservoirs in Hungary.



A hőáramsűrűség eloszlása a Pannon-medencében és környezetében. Az izovonalak egysége mW/m².
(Dövényi, 2006)

This heat flow map was approximated (and constructed) by T measurements in 1500 boreholes. There is a thinner lithosphere in the Great Plain, for this reason elevated (80-100mW/m^2) heat flow can be observed. The presence of karst water results in cooling effect.



Dövényi et.al. (2006)

Convection is favoured over conduction in geothermal pojects (why?)

Make a (theoretical) comparison ***between*** **the heat gained by conduction** assuming

a terrestrial heat flow of $90 mW/m^2$ if the utilized surface is $9 km^2$ ***and*** **the heat provided by a thermal well** with flow rate of 400l/min if the temperature of the out flowing water is 40 ^0C (average annual temperature is 10 ^0C, the specific heat capacity of the water is 4183J/kg K).

The heat output gained by conduction is

$$\mathbf{90\ mW/m^2 * 9km^2 = 0.81MW},$$

and the ***heat output gained by convection*** is

$$\mathbf{4183\ J/(kg\ K) * 400kg/60sec * 30K = 0.837MW}.$$

Radioactive heat production

In the course of radioactive decays energetic particles and gamma rays are emitted. The mass is converted to heat. Only abundant isotopes with significant half-life - comparable to the age of the Earth – can be considered as almost continuous heat source. The most relevant isotopes that fulfil these conditions are ^{238}U, ^{235}U, ^{232}Th, ^{40}K. The most important particles are helium nuclei (positively charged alpha particles) and electrons (negatively charged beta particles). Negative beta decay can be experienced most frequently (46%), and the ratio of electron capture is also relatively high (25%). The occurrence of additional nuclear disintegrations is relatively low: positive beta decay 11%, alpha decay 10%, and spontaneous fission of heavy isotopes 8%. Despite the low occurrence of alpha disintegration, 90 % of the total heat produced by radioactivity is due to the alpha decays. In the case of same amount from ^{238}U and ^{235}U, the heat produced by the decay chain of ^{235}U is greater than that of ^{238}U. The heat production of ^{40}K is due to negative beta disintegration and the gamma radiation of KEC.

For unit mass of these unstable isotopes heat generation rate can be given. These values are 95.2*10^{-6} W/kg, 25.6*10^{-6} W/kg, 0,00348*10^{-6}W/kg for U, Th, K, respectively. In the knowledge of concentration (C, in ppm), the heat Q_r produced by radioactivity in a rock can be calculated:

$$Q_r = 95.2C_U + 25.6C_{Th} + 0.00348C_K$$

Radioactive heat production

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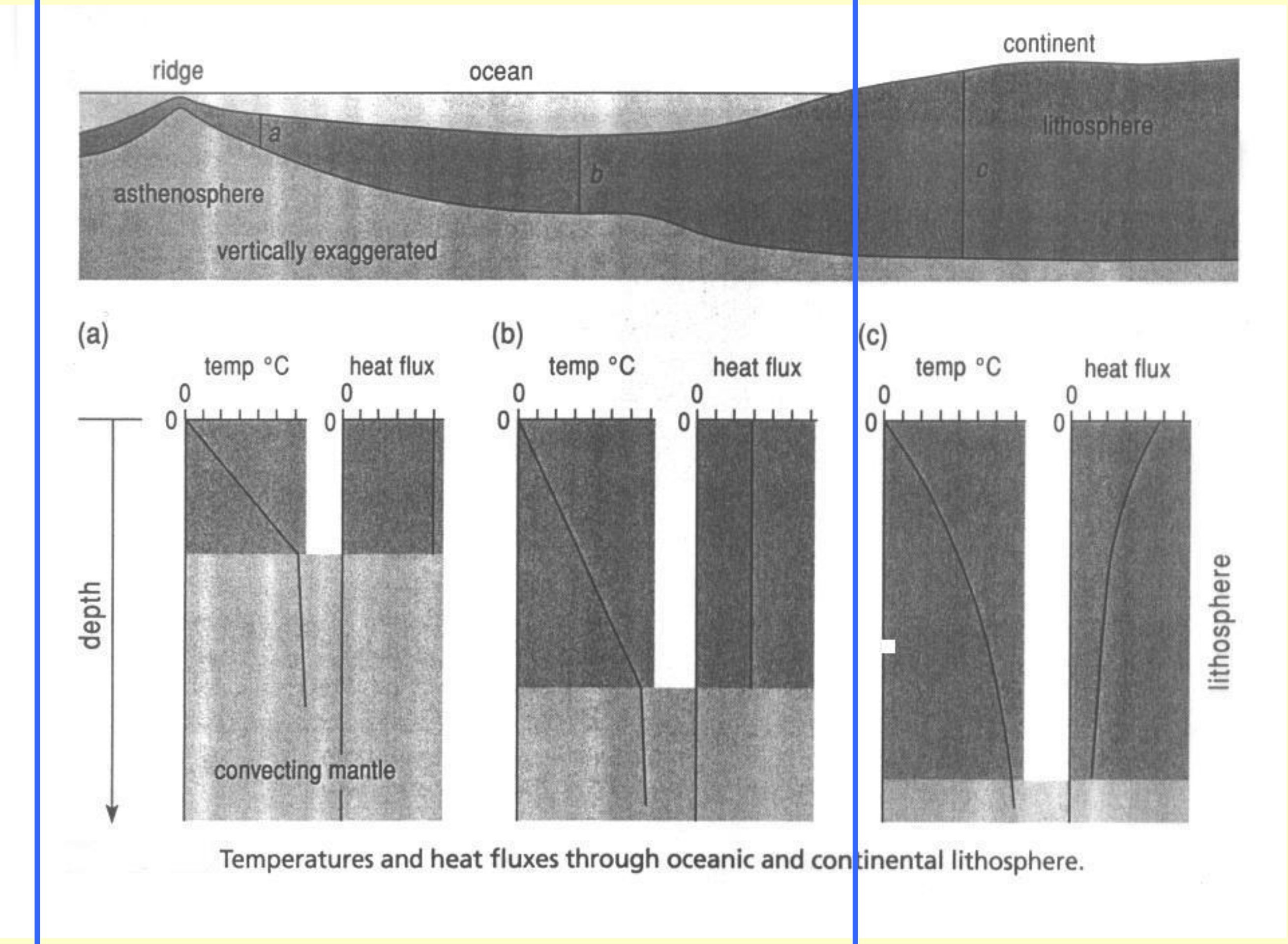
$$Q_r = 95.2C_U + 25.6C_{Th} + 0.00348C_K$$

For a granite with 4.6ppm U, 18ppm Th, 33000ppm (3.3%) K the heat produced by radioactivity is 1.01 10^{-9} W/kg

The depth dependence of heat flow and temperature for an oceanic crust



Temperatures and heat fluxes through oceanic and continental lithosphere.

At the bottom of the oceanic crust constant temperature -1300 ^{0}C- can be assumed. The temperature at the sea-bottom is 0 ^{0}C, and for the oceanic crust a constant thermal conductivity value (λ) is assumed. *There is no granite in the oceanic crust, for this reason there is no radioactive heat production.* The heat flow originates from the mantle has to be constant in the function of depth, however it depends on the thickness of the oceanic lithosphere.

$$\vec{q} = -\lambda gradT = -\lambda \vec{G}$$

The less the thickness of the oceanic lithosphere is, the greater the geothermal gradient and the heat flux must be.

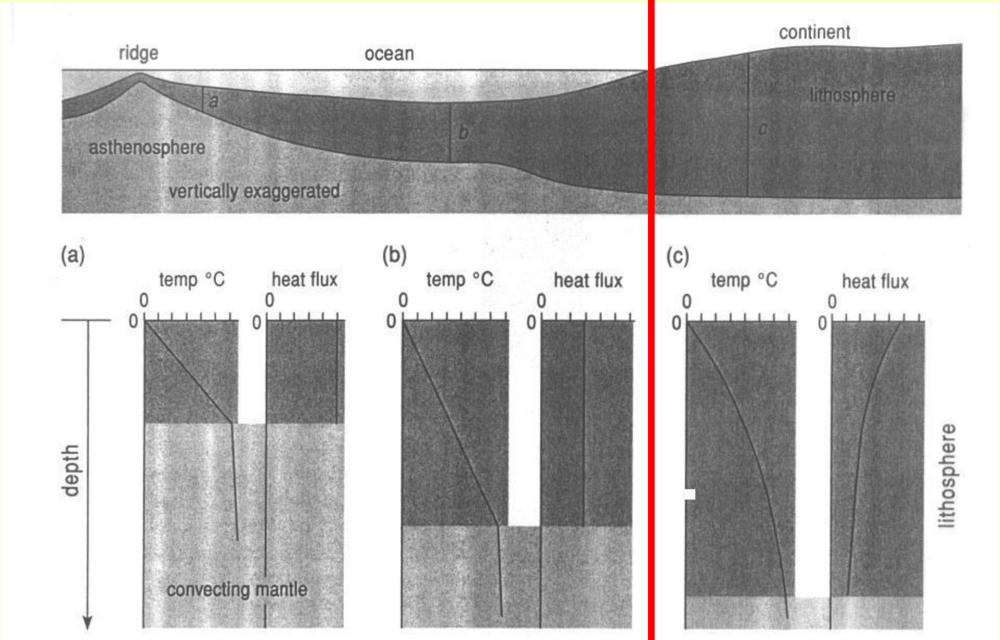
Mussett, Khan 2000

The depth dependence of heat flow and temperature for a continental crust

Mussett, Khan 2000

At the bottom of the continental lithoshpere the average heat flux (q_m) is 28-32 mW/m^2. The average continental heat flux (q_0 at the surface) is 65mW/m^2. At the surface the sum of the heat flow coming from the mantle (q_m) and the heat produced by radioactive decays in the crust can be measured:

$$q_0 = q_m + \rho z_r S_0$$



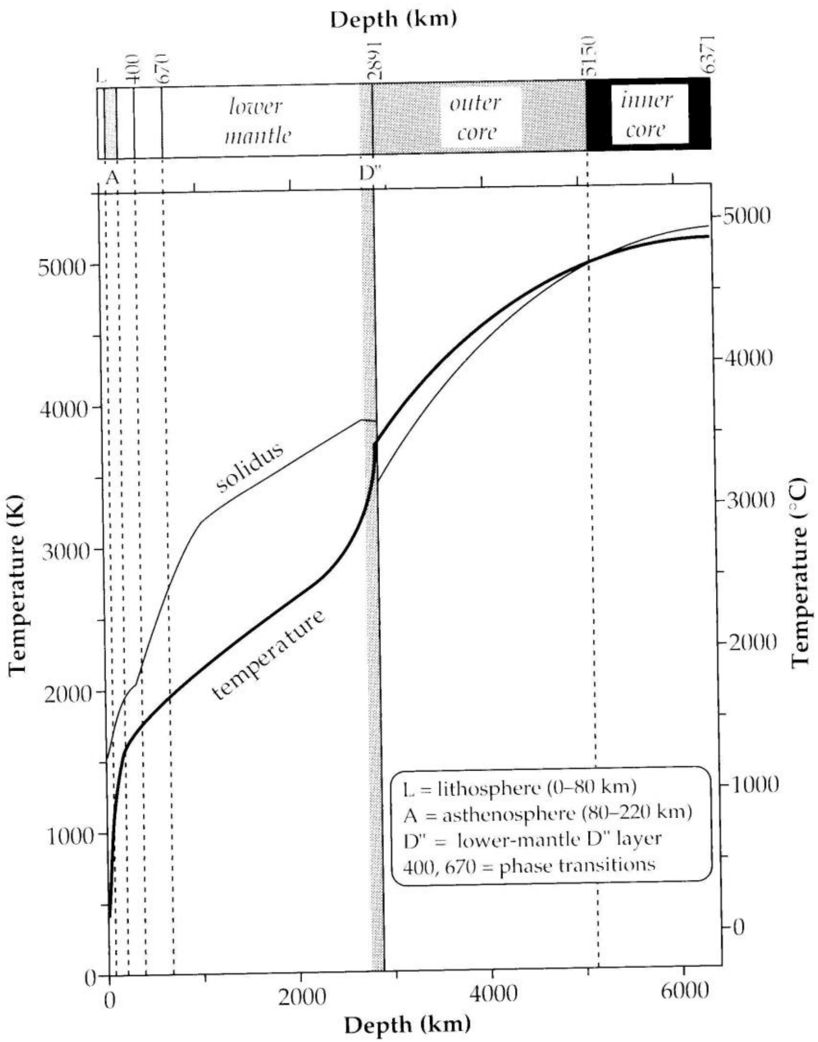
Temperatures and heat fluxes through oceanic and continental lithosphere.

The temperature-depth relationship if the heat produced by radioactivity is decreased exponentially with depth:

$S=S_0 \exp(-z/z_r)$, **where z_r denotes the depth at which the surface value decreased to 1/e.**

$$T = T_0 + \frac{q_m}{\lambda} z + \frac{(q_m + q_0)}{\lambda}(1 - e^{\left(\frac{-z}{z_r}\right)})$$

Earth's age, thermal and electrical properties



Variations of estimated temperature and melting point with depth in the Earth (based upon data from Stacey, 1992).

Assumed temperature distribution in the function of depth

Convection in the mantle

Mantle can be considered to be rigid, because it supports both P and S waves propagation. It has also high viscosity, however the time scale of geological processes is so long that long-term flow may occur in it. This flow is a thermally driven convection. It takes place in it, because the buoyancy force is significantly greater than the diffusive-viscous force. The ratio of the two forces is expressed by the Rayleigh number. If it exceeds a critical value, the convection develops. The thermal convection of the mantle influenced by the position of thermal boundaries. Due to the relatively small value of Re number there must be a laminar flow.

$$R_a = \frac{g\rho\alpha\Delta T}{\kappa\eta} D^3$$

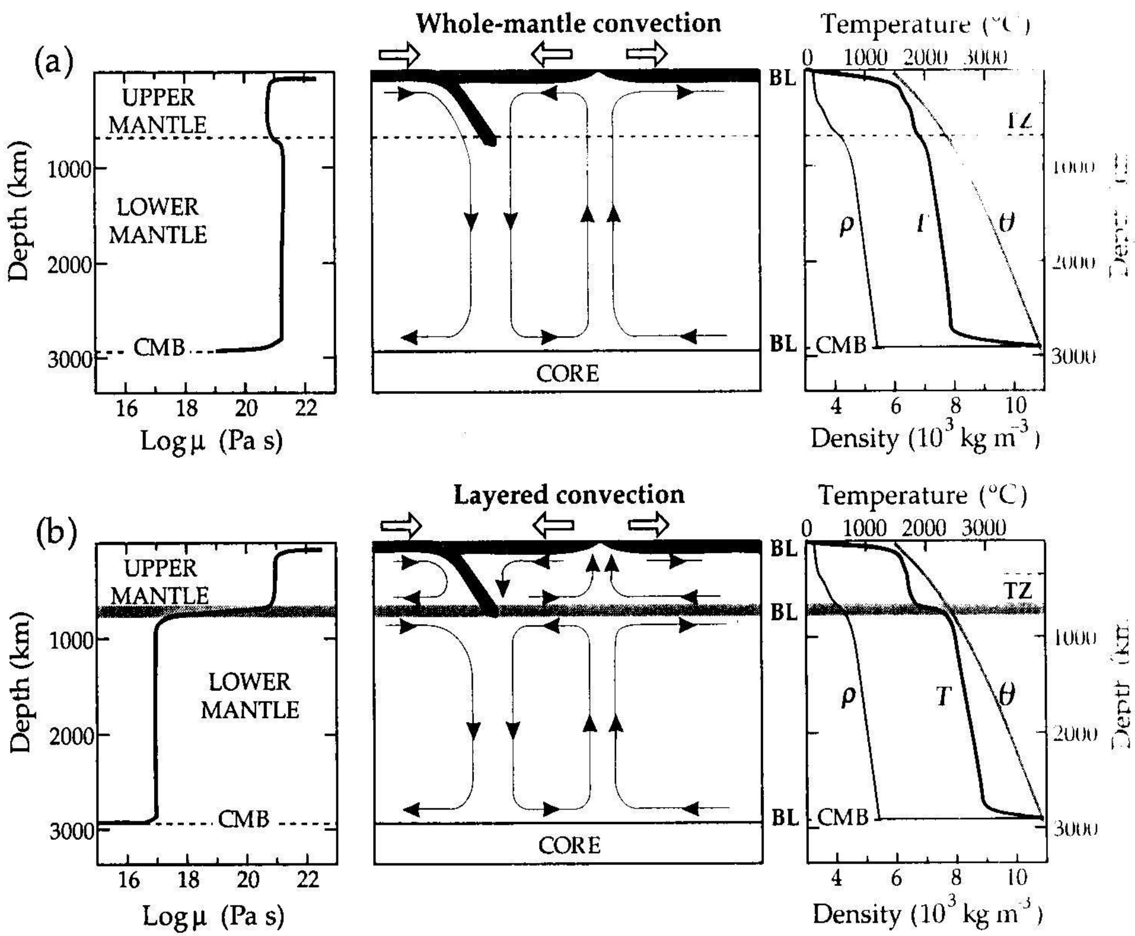
Some physical parameters for mantle convection models (mostly from Jarvis and Peltier, 1989)

The critical Rayleigh numbers (Ra_c) for the onset of convection in each part of the mantle are calculated assuming a superadiabatic temperature gradient $\theta = 0.1$ K km^{-1} and a mean gravity $g = 10$ m s^{-2}. Lower mantle parameters are interpolated from the upper- and whole-mantle values.

Physical parameter	Units	Upper mantle (70–670 km)	Lower mantle (670–2890 km)	Whole mantle (70–2890 km)
Layer thickness (H)	km	600	2220	2820
Expansion coefficient (α)	K^{-1}	2×10^{-5}	1.0×10^{-5}	1.4×10^{-5}
Density (ρ)	kg m^{-3}	3700	5500	4700
Specific heat (c_p)	J kg^{-1} K^{-1}	1260	1260	1260
Thermal conductivity (k)	W m^{-1} K^{-1}	6.7	20	15
Thermal diffusivity (κ)	m^2 s^{-1}	1.4×10^{-6}	3×10^{-6}	2.5×10^{-6}
Dynamic viscosity (η)	kg m^{-1} s^{-1}	1×10^{21}	2.5×10^{21}	2×10^{21}
Kinematic viscosity (ν)	m^2 s^{-1}	2.7×10^{17}	4.5×10^{17}	4.3×10^{17}
Rayleigh number (Ra_T)	—	7000	180,000	820,000

Convection in the mantle

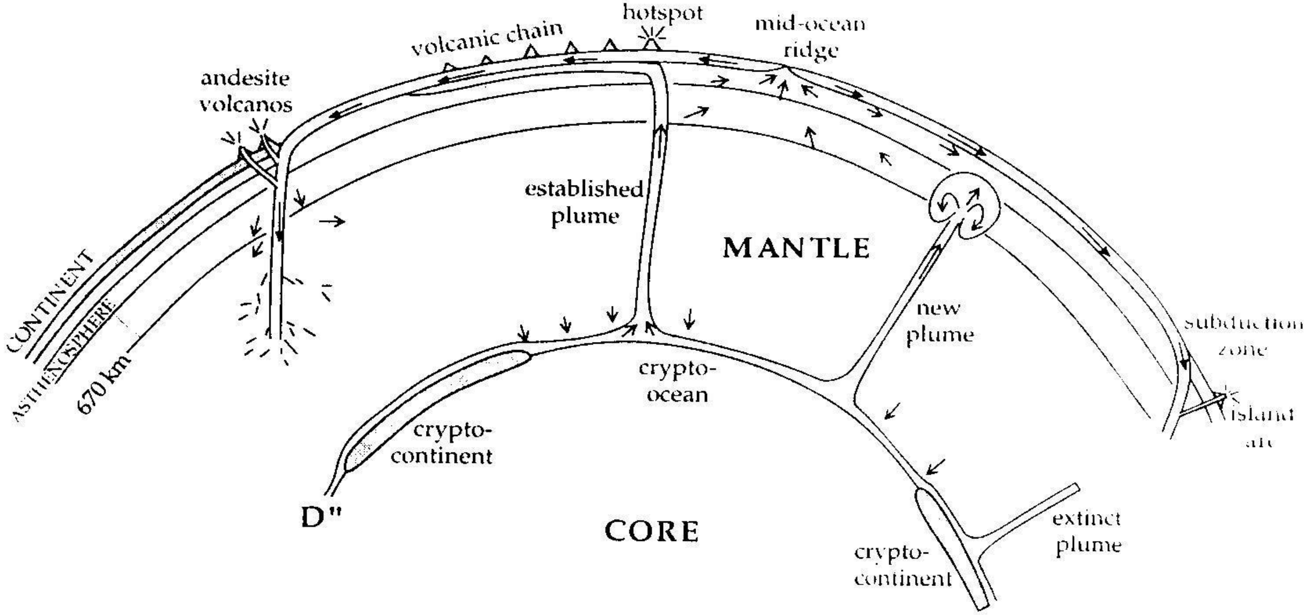
Possible convection flow pattern (*center*) and profiles of viscosity μ (*left*), and density ρ, temperature T and solidus temperature θ (*right*) for (a) whole-mantle convection and (b) layered mantle convection. TZ is the upper-mantle transition zone, BL are boundary layers, CMB is the core–mantle boundary (based upon Peltier *et al.*, 1989).



Whole-mantle and layered convection pattern can be seen above. **Source: Lowrie, 2000.**

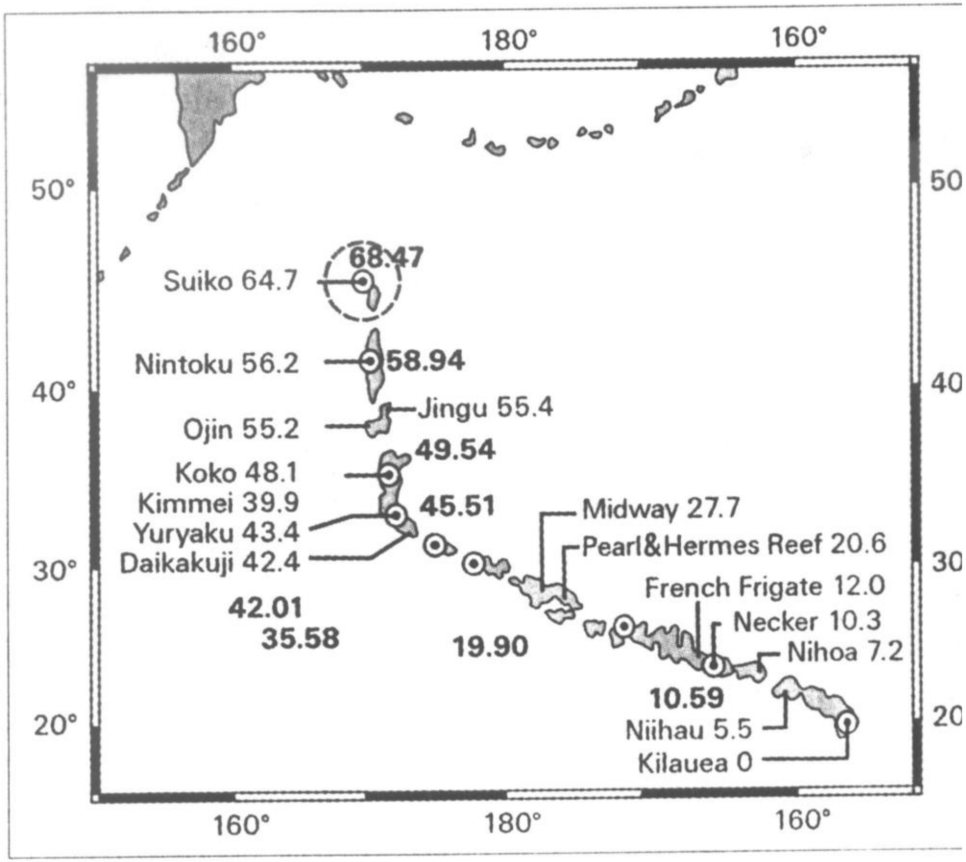
Mantle plumes

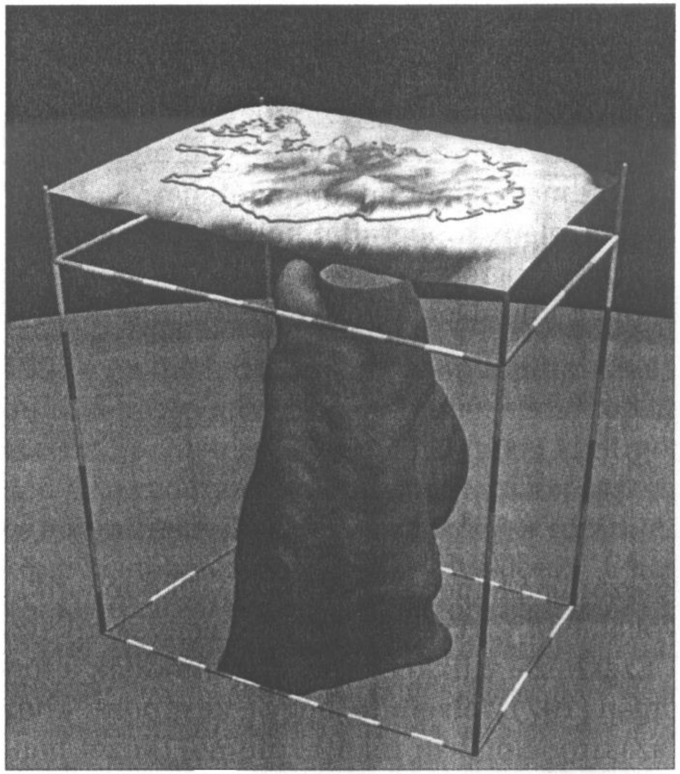
An idealized cross-section through the mantle, showing convective flow and the relationship of mantle plumes to the D"-layer (after Stacey, 1992).



Convection is dominant over conduction in the mantle. Mantle plume is an upwelling, low viscosity hot magma which penetrates into the mantle and it frequently reaches the surface. The plumes are assumed to have fixed position for a long time. They must have important role in plate tectonic motions.

Mantle plumes





Izland alatt található hőoszlop tomográfiai módszerrel meghatározott formája. (Wolfe et al. 1997)

The same mantle plume with fixed position produced significant volcanic activity and the hot spot in Hawaii. Its total energy is 2300GW (Kis K. 2007), left. Seismic tomography proved that the source of the heat in the case of Island can be at the CMB, on the right.

Thermal conductivity measurement in lab

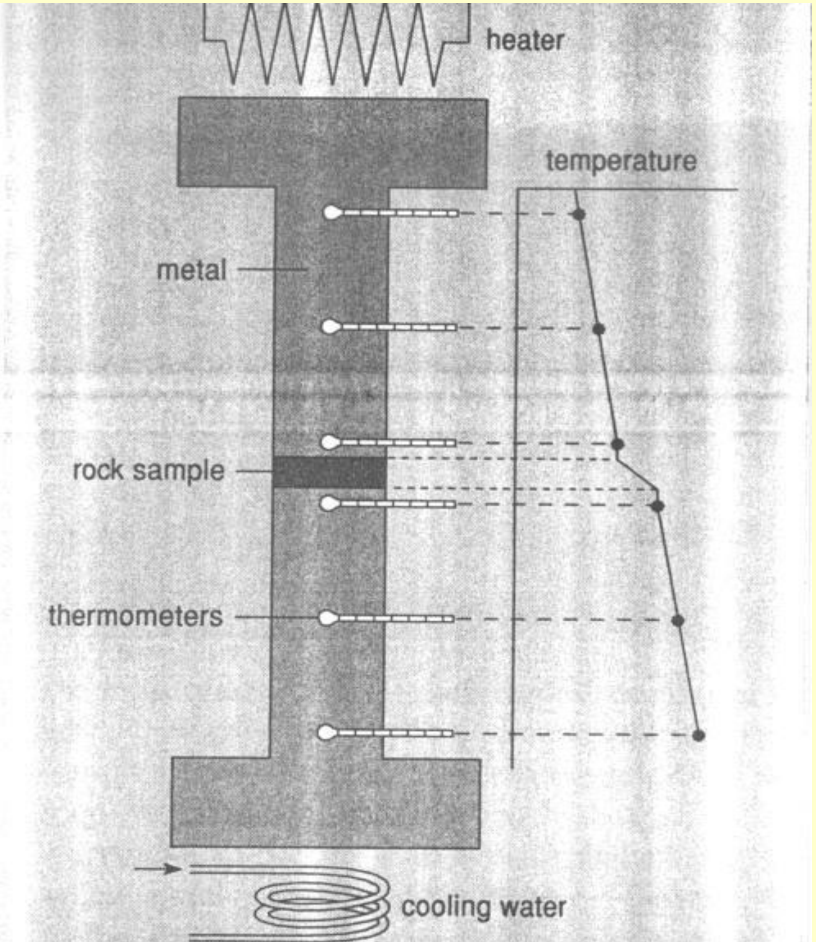


Figure 1 Measurement of thermal conductivity in the laboratory.

$$q = \lambda gradT$$

The core is sandwiched between two metal bars with temperature difference. The same amount of heat flows through the core as in the bars in vertical sense (because the system is insulated). In the knowledge of thermal conductivity of the metal and the temperature values (at least four values to measure the thermal gradient for the metal and the rock, respectively) the thermal conductivity of the core can be calculated.

Mussett and Khan, 2000

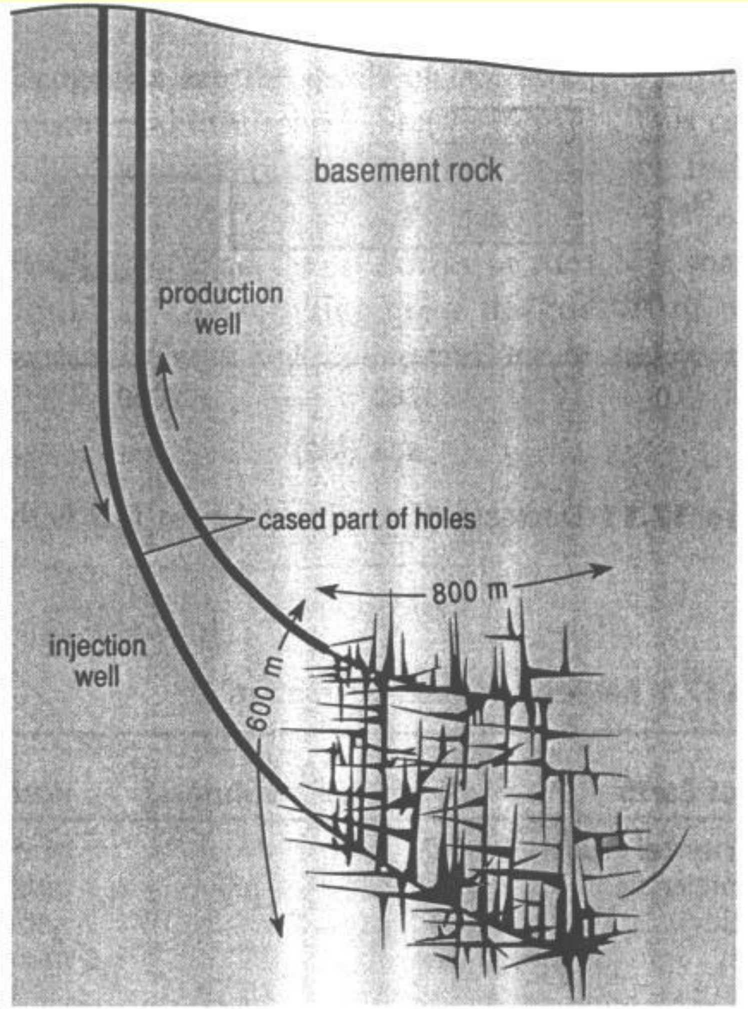
$$q = -\lambda gradT = -\lambda G$$

ANYAG	FAJLAGOS HŐVEZETŐKÉPESSÉG λ (W/m ^{0}C)
Andezit	1.35-4.86 (2.26)
Bazalt	1.12-2.38 (1.69)
Diabáz	2.1-2.3 (2.2)
Gabbro	1.98-3.58 (2.57)
Diorit	2.02-3.33 (2.50)
Granodiorit	2.0-3.5 (2.63)
Gránit	2.3-3.6 (3.07)
Kősó	5.3-7.2 (5.7)
Száraz homok/agyag	0.2-0.4
Nedves agyag	0.8-1.5
Nedves homok	1.1-2.1
Megművelt talaj	0.2-1.2
Víz	0.6 (25 ^{0}C)

Geothermal energy provided by nature

Fig. 10.4 Thermal gradient map of Milos island, Greece. The main geothermal areas (shaded zones) are characterized by large values of thermal gradient (>8° C/10 m). Open circles show locations of boreholes with large steam production. (After Fytikas, 1977.)

Geothermal energy



Hot, dry rock heat-extraction system.

Enhanced geothermal system (EGS)

Developed from Deep Heat Mining (DHM) and Hot Dry Rock (HDR) technologies where hydraulic fracturing is applied.

Injection and producing wells.

200 ^{0}C

Mussett and Khan, 2000

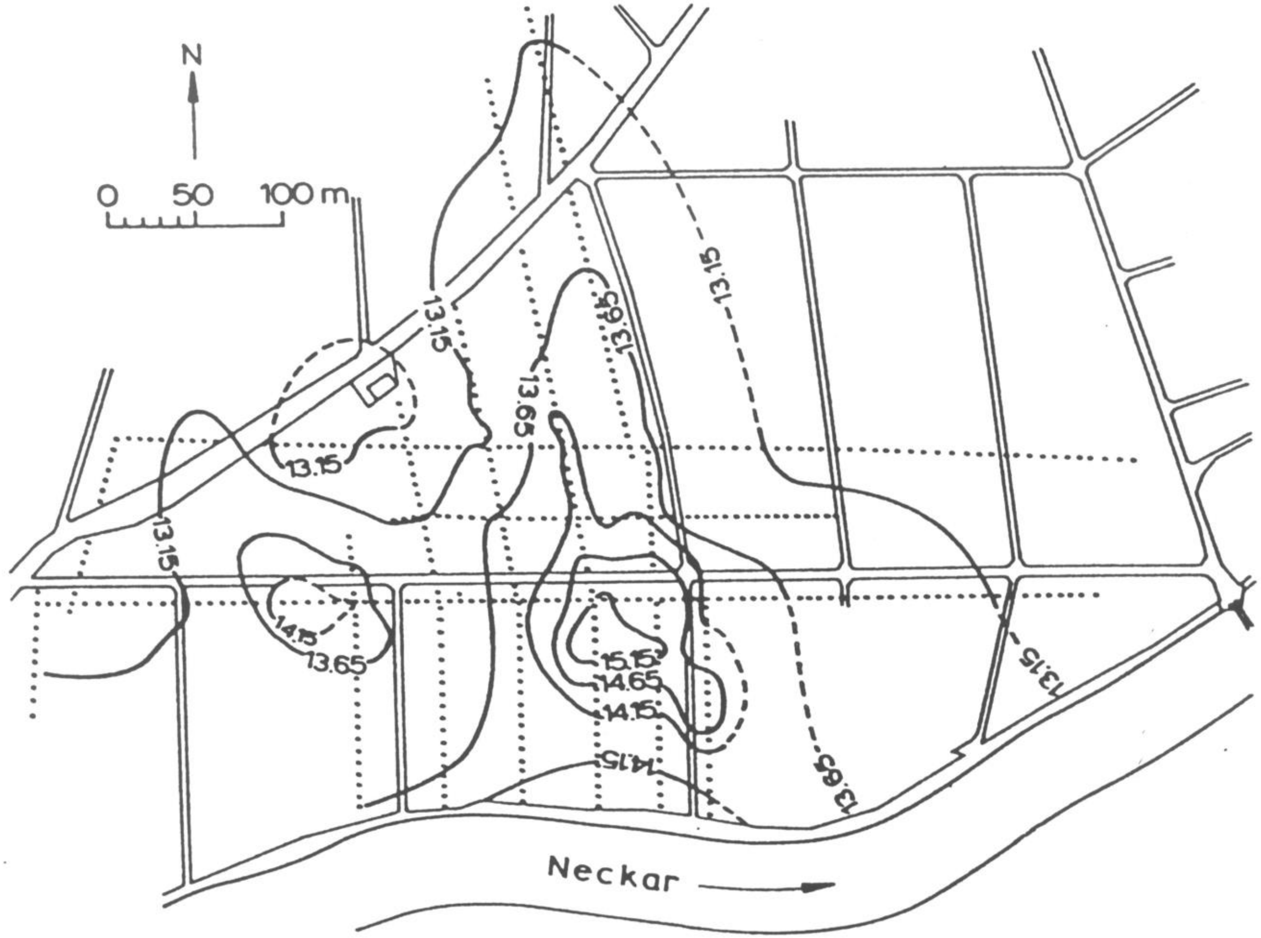
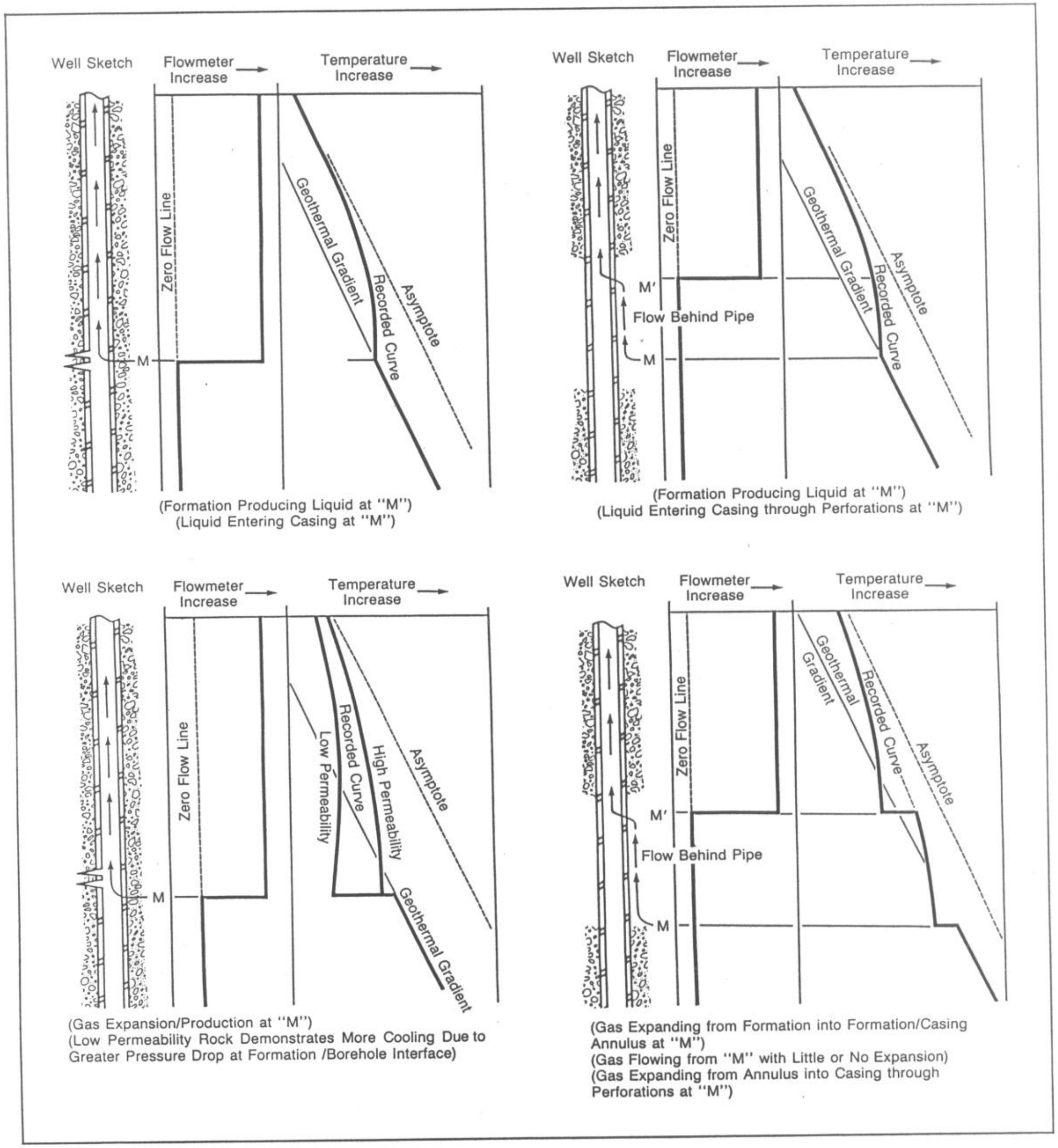


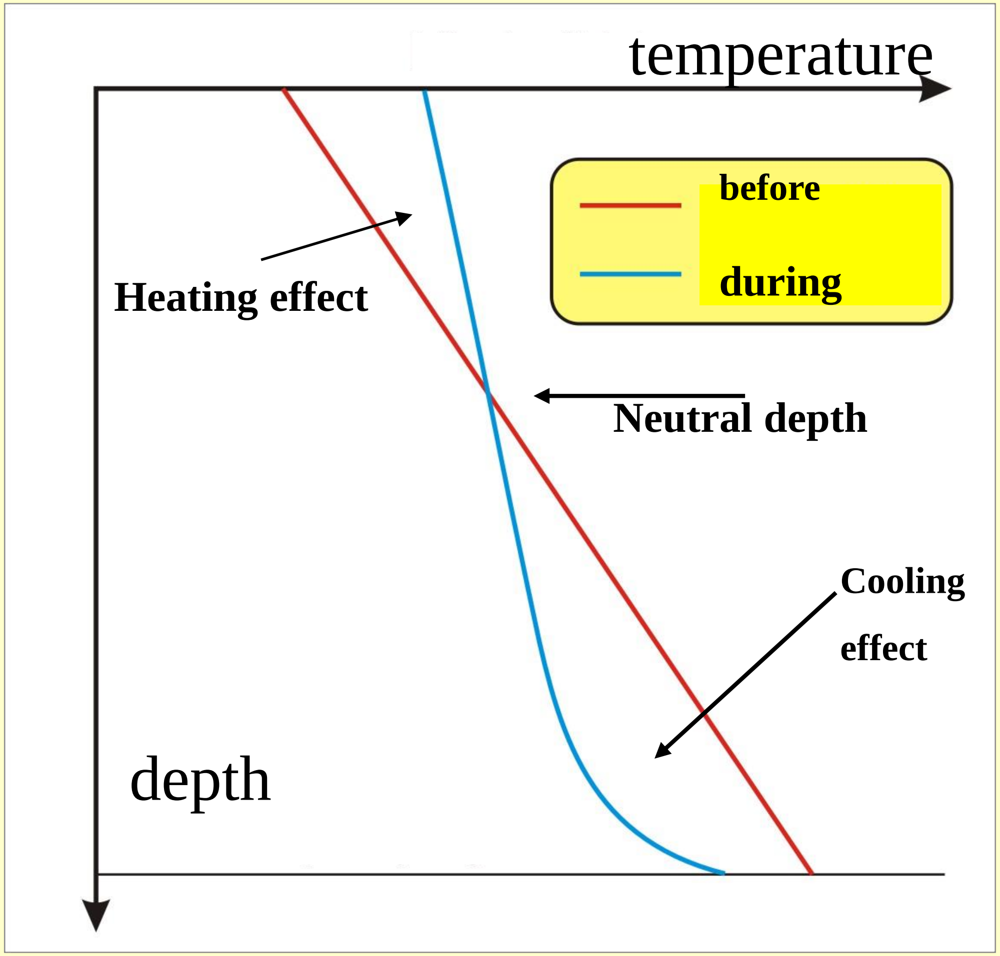
Fig. 10.8 Temperature anomaly at 1.5 m depth in Kiebingen, Neckar Valley (southern Germany). Contour interval is 0.5 °C. The anomaly is caused by karst water which emerges from some hundred meters depth into the groundwater. (After Kappelmeyer and Haenel, 1974.)

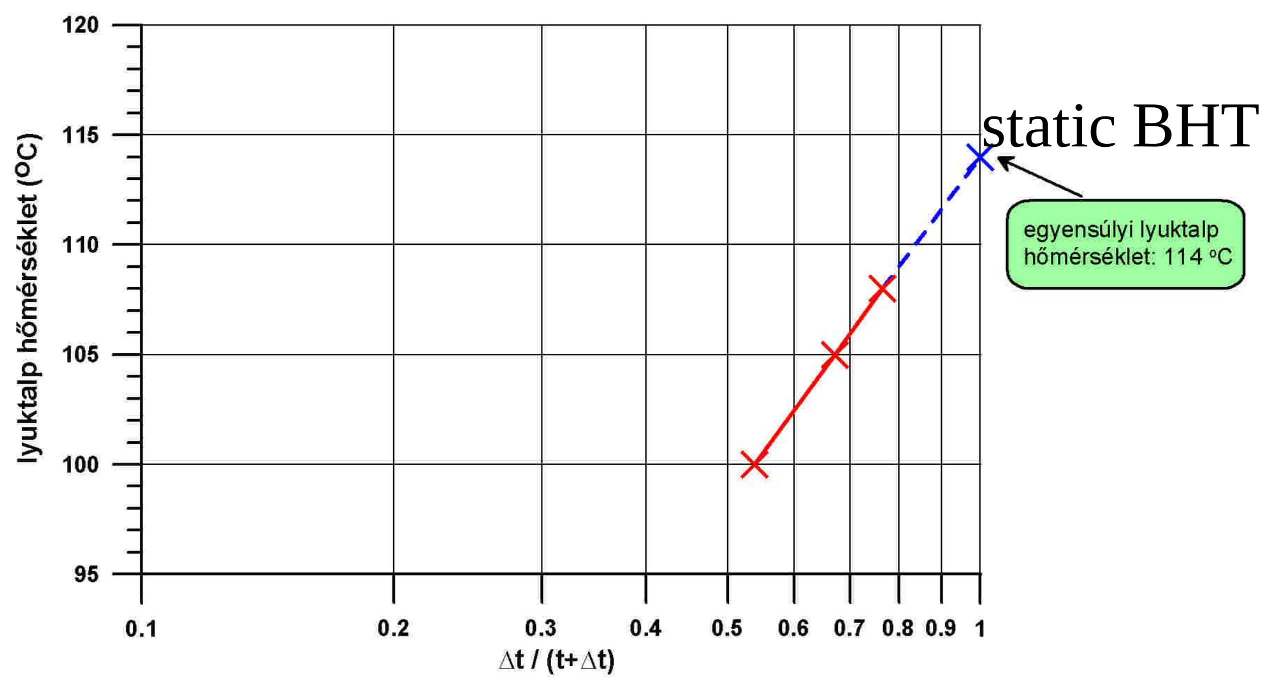


-Temperature log responses

Schlumberger

Temperature distribution in a vertical borehole

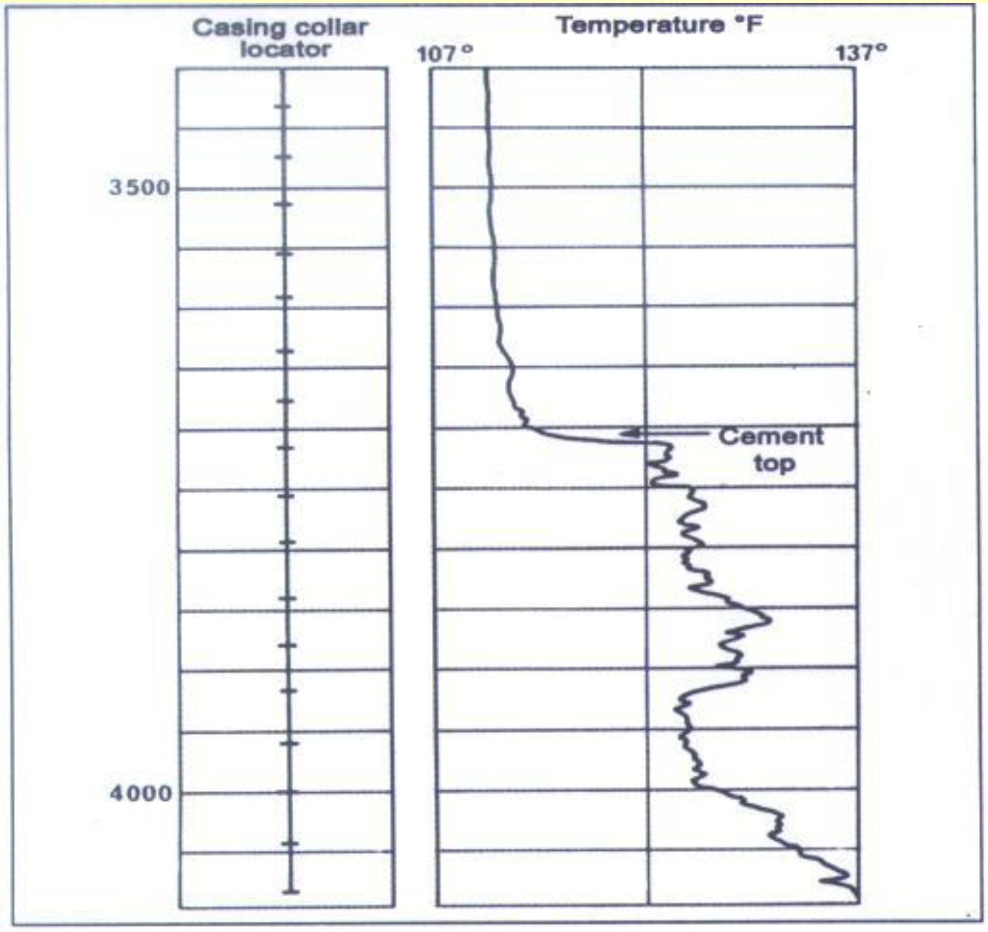




mérés	Δt (óra)	Δt / (t+Δt), (t=6 óra)	mért hőmérséklet 3200 m-nél (°C)
M_1	7	0,538	100
M_2	11,5	0,671	105
M_3	19,5	0,765	108

If we measure at least two times the bottomhole temperature after circulation, then we can determine the static BHT by extrapolation applied in a semilog coordinate system. Here t denotes the circulation time of drilling mud, $\triangle$t stands for the elapsed time after circulation, and the measured bottomhole temperature is plotted along the vertical axis (Fertl& Wichmann ,1977).

Detection of cement top



Detection of the cement height behind casing from a temperature survey (courtesy of Schlumberger).

Questions

- What are the most important ways of heat transfer?
- What do the heat flow lines and the isothermal lines present?
- What do the Fourier-equations state?
- What do you know about the heat convection?
- What is the essence of radioactive heat production?
- Is there any difference between the depth dependence of heat flow and temperature for an oceanic crust and for a continental crust? If there is what is the reason of it?
- How can the thermal conductivity be determined in lab for a core sample?
- How can you make difference between gas and fluid entry into a tubing /casing if you measure both the flow rate and the temperature in the function of depth?
- How can we locate the cement top (in the annulus) by temperature log?
- How can you determine the static BHT?