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**Theses of Doctoral (Ph.D.) Dissertation**

# MODERN INVERSION METHODS FOR THE INTERPRETATION OF WELL-LOGGING DATA

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# I. Scientific background and aims

Open-hole well-logging measurements play an important role in the mineral exploration. One can get almost in-situ information by means of several kinds of well-logging probes with different vertical resolution from the near vicinity of the borehole. Data collected this way can be related to several physical properties of the rocks situated in the upper part of the Earth crust. By the interpretation of well-logging data it is possible to determine some geometrical (e.g. layer-thickness, formation dip) and petrophysical (porosity, water saturation, shale content, specific volume of rock matrix, permeability) parameters of geological formations that give us essential information about the spatial position of the structure and the quantity of the useful mineral resources. The professional practice lays ever-increasing claim to the quality of the well log interpretation results all over the world. It is particularly true for the hydrocarbon exploration, where petrophysical and geometrical parameters have to be derived from logs for the calculation of oil and gas resources beside more and more complex geological situations.

In the industrial practice the well-logging interpretation methods are mainly based on deterministic procedures at present. Besides these statistical methods using largely geophysical inversion techniques have appeared since the eighties. The essence of the industrial applications is that they perform local gradient-based linearized optimization. It can give information not only for the petrophysical parameters but their accuracy and reliability. These methods are widespread because of their quickness. However their searching technique is successful only in that case when we start the optimization process not too far from the solution in the parameter space. Otherwise, it can be shown that the given result is not always the optimal one. The background of this problem is that the linearized methods tend to assign the solution of a local minimum of the objective function instead of the absolute one. To cope with this problem, global optimization methods are used. Despite of some little reference in literature, global optimization methods can be effectively applied in well-logging. Moreover, the simple structure of the well-logging forward problem serves the application of these random searching methods in the point of view of computational quickness. In consequence, petrophysical parameters estimated by the inversion of well-logging data can be determined more accurately, which leads to an improvement in the quantitative characterization of rocks containing mineral resources.

To solve the nonlinear geophysical well-logging inverse problem point by point inversion methods are conventionally used that utilize the data set of a certain depth-point to determine the petrophysical model parameters only to that point. Having relatively few data in the point the number of the parameters to be determined is fewer than what the inverse problem would demand. Namely, the theoretical probe response functions underlying the direct problem contain not only the unknown petrophysical parameters but a large number of textural and zone parameters that can be assumed as invariable quantities in order to preserve the over determination of the inverse problem. Since the number of the measured data is slightly more than the number of the unknowns, the marginal over determination causes relatively low accuracy and reliability in parameter estimation. In addition to it, the layer-thicknesses that do not appear in the local response functions can not be computed by point by point inversion.

In my thesis solving the problems arose from the defects of quality of the conventional point by point inversion method, a new interpretation method was introduced. Within the frame of the so-called interval inversion method the data set of a greater depth-interval was integrated in one inversion procedure that resulted in a largely over determined inverse problem for the unknown petrophysical parameters. The great over determination ensures high accuracy and reliability in parameter estimation furthermore it supports the introduction of further parameters into the inverse problem that have not been treated as unknowns in well-logging inversion so far. The development of the interval inversion method is the result of the research work of the Inversion and Tomography Research Team of the Department of Geophysics, University of Miskolc. I joined to the research team in 1999 with my MSc thesis titled was "The analysis of the point by point and interval inversion of well-logging data by means of linearized and global optimization methods".

In case of interval inversion method we have to extend the validity of local response functions for the whole depth-interval processed. The proper response functions for the direct problem were proposed first by M. DOBRÓKA in 1995. The discretization of these depth-dependant response functions can be carried out in different ways. The method used in the thesis based on the series expansion of the depth-dependant model parameters by means of known basis functions. Earlier in my MSc thesis I studied the series expansion method set up of unit step functions on layer-wise homogeneous geological model. But the basis functions can be chosen independently and suited to the actual geological situation. In my PhD thesis besides the unit step functions I demonstrated the series expansion method by power basis functions, and an interval inversion procedure with unknown layer-thicknesses as a new approach in well-logging inversion.

## II. DESCRIPTION OF THE RESEARCH WORK

After reviewing the literature I worked out the methodology of the new inversion procedures for the interpretation of well-logging data. Within the frame of this I introduced some further developed point by point, and the so-called interval inversion method based on new principles. The new inversion methods were algorithmized, thereafter a well-logging inversion software package was developed in MATLAB 6 system including 297 items of script files and function modules. Using the software I compared on the one hand point by point inversion methods with the interval inversion ones, on the other hand linearized optimization methods with global ones numerically. At first the inversion algorithms were tested by means of noisy synthetic well log data, and then were used to the interpretation of real data measured in a borehole. Overall ten new inversion algorithms were studied by means of nine synthetic and measured data sets and five inversion models were subjected to the experiments. With the research I requested an answer for the accuracy, the reliability, the stability of the inversion methods. Besides the advantages of the parameter determination by the new methods against the conventional ones were shown.

At first the traditional and the newly developed point by point inversion methods were tested by means of noisy synthetic data. A known petrophysical model was used for the calculation of quasi measured well-logging data, which were inverted to inform me about the accuracy, the stability of the linearized (DLSQ-P) and global (MMSA-P and FGA-P) inversion methods (the estimation errors were also computed in case of linearized optimization). The investigations covered some questions put usually in optimization theory in particular the number of iteration steps and CPU time. An additional data set containing outliers was also inverted, and the convergence velocity of global optimization was improved as well.

Linearized (DLSQ-I and LAD-I) and global (MMSA-I and FGA-I) interval inversion methods were tested using noisy synthetic data. I made comparison between point by point and interval inversion procedures. Layer-wise homogeneous model was applied for the series expansion giving depth-dependent petrophysical parameters that were computed by interval inversion beside fixed layer-boundary coordinates. Numerical experiments also covered the computation of relative data and model distances, run time of the algorithms, development of the convergence related to iteration steps, and the estimation errors and correlation coefficients. At last in case of a data set containing outlying noise resistant global inversion was achieved.

Above interval inversion experiments were also performed for unknown layer-thicknesses. To determine the thickness information global inversion procedures (as MMSA-H and FGA-H) were applied. The stability, accuracy and the convergence velocity were studied in case of a data set superposed by purely Gaussian noise and then outliers as well.

Interval inversion method was extended for the case of the determination of model parameters undergoing a vertical change within an arbitrary depth-interval (e.g. in layer). For this goal a series expansion using power basis functions were implemented, where the series expansion coefficients of a chosen layer were collected into the model parameter vector among the other unknowns. In this synthetic problem I got a solution to determine the series expansion coefficients without the knowledge of the homogeneous model (each coefficient is unknown in the series expansion) by stable and initial model independent global optimization.

Since the global optimization methods have rather slow performance in case of large number of unknowns, and unable to give a value of estimation errors and any information about the reliability of the results from one program run, therefore I developed a combined interval inversion method. It is based on the sequential application of global and linearized optimization methods. In fact, starting with a global searching method we can get to the near vicinity of the solution independently from the initial model, from where switching over to linearized optimization the optimum can be obtained very quickly. This method also allows us to characterize the error and reliability of the estimated parameters through the elements of the computed covariance and correlation matrix. A Genetic Algorithm based on a float-encoded representation was chosen for the global optimization because of its remarkable adaptation facility and robust behaviour.

I was concerned with a question of the well-logging forward modelling, which gave me valuable information for the inversion experiments as well. Namely, I analyzed the parameter-sensitivities of the theoretical sonde response functions with respect to the accuracy of inversion parameter estimation. Parameter sensitivity functions inform us about the rate of influence of model parameters on data in a certain parameter range. For this purpose I computed the sensitivity functions of the main type of well logs related to porosity, water saturations, shale and sand content on a four-layered model and interpreted the results. Comparing two models the connection between the sensitivity values and the accuracy of the determination of unknowns was proved in case of linearized point by point inversion. I drew a conclusion from these experiments that unknowns showing small sensitivity in a given parameter range have to be treated as fixed values (only in case of having satisfactory a priori information) so as to increase the over determination of the inverse problem and improve the quality of the estimation.

Employing the experience of the synthetic experiments, measured well-logging data were interpreted by the new inversion methods. The well logs originated from a Hungarian hydrocarbon exploratory borehole. At first point by point inversion methods and then interval inversion procedures were executed. Using the latter one automatic layer-thickness determination was carried out as well. In a gas-bearing layer I estimated the porosity, the shale content and the water saturation (movable and irreducible gas saturation) using an interval inversion algorithm based on a series expansion with power functions. These results were compared to deterministic ones computed by log analysts at the Petrophysical Department of the Hungarian Oil and Gas Company.

## III. NEW SCIENTIFIC RESULTS

### 1. Development of point by point well-logging inversion methods using global optimization techniques

Global point by point inversion algorithms were developed in order to increase the accuracy and reliability of the estimated petrophysical parameters interpreted by conventional (industrial practice used) inversion methods. The global Simulated Annealing method based on Metropolis algorithm was modified with a new energy function that ensures faster convergence to the solution than the classical one.

The new point by point inversion methods as the Modified Metropolis Simulated Annealing (MMSA-P) and the Float-encoded Genetic Algorithm (FGA-P) were subjected to numerical experiments on noisy synthetic well log data. It was demonstrated that more accurate and reliable petrophysical parameter estimation can be attained by the new methods than by the linearized point by point inversion. It was also shown that in case of initial models characterized by large model (and data) distance and high correlation between model parameters, the MMSA-P procedure assures faster convergence to the solution compared to Metropolis algorithm.

### 2. Implementation of interval inversion of well-logging data in case of known layer-boundary coordinates

Interval inversion algorithms based on linearized (DLSQ-I, LAD-I) and global optimization (MMSA-I and FGA-I) were developed for the interpretation of well-logging data. Interval inversion allows us to interpret all the data of a given depth-interval jointly in one inversion procedure.

It was demonstrated that the accuracy of parameter estimation in case of interval inversion can be increased at least by one order of magnitude compared to point by point inversion in case of layer-wise homogeneous geophysical model.

## 3. Implementation of interval inversion of well-logging data assuming petrophysical parameters showing vertical changes in depth

A new interval inversion algorithm was developed that supports the determination of petrophysical parameters varying vertically in an arbitrary depth interval. Power functions were used for the discretization of model parameters in a series expansion.

A combined interval inversion algorithm was applied based on the joint use of a linearized and a global optimization method as FGA+DLSQ-I. By this technique, the convergence velocity of FGA algorithm was accelerated by one order of magnitude. Furthermore, the parameter estimation errors were also computed by means of the combined inversion algorithm (global optimization methods have not been provided with it in case of a single inversion program run so far).

## 4. Implementation of interval inversion of well-logging data in case of unknown layer-boundary coordinates

A new interval inversion algorithm was developed that gives an estimate for the layer-thicknesses and the petrophysical parameters simultaneously in one inversion procedure. Global optimization methods were implemented to form MMSA-H and FGA-H algorithms.

It was shown that the inversion for the layer-thicknesses can be solved in stable numerical procedure, which means a new approach in geophysical well logging inversion. Moreover an appropriate selection of energy function was made for MMSA-H algorithm that resists the outliers of the well log data set and gives a quick convergent solution.

## 5. Interpretation of measured well-logging data by interval inversion

As a practical application the new inversion methods detailed in the former theses were applied to the interpretation of a well log data set measured in a domestic well bore for hydrocarbon exploration. I performed interval inversion for layer-characteristic petrophysical parameters and layer-thicknesses presuming layer-wise homogeneous geophysical model. A proposal was made for the application of the interval inversion method as a possible technique to classify the hydrocarbon reservoirs quantitatively.

At last a combined interval inversion algorithm (FGA+DLSQ-I) was applied to estimate petrophysical parameters varying vertically in a hydrocarbon reservoir. It resulted in an improvement for the results of series expansion based on layer-wise constant functions.

## PRACTICAL APPLICATION OF THE RESULTS

The efficiency of the new inversion methods were shown in the thesis. It was concluded from the results of the synthetic tests and the evaluation of well-logging measurements that the global interval inversion method based on automatic layer-thickness determination is the most powerful that can be proper to apply in the industrial practice simultaneously besides the deterministic methods of long standing. Interval inversion is independent from the initial model and very stable. It results in very accurate and reliable estimation as well. Moreover, it can give information about the position of the layer-boundaries in the borehole, which can not be realized by any conventional point by point inversion method.

The interval inversion method opens the door to further possibilities. It can be developed to treat some textural and zone parameters as unknowns and determine their values automatically like as the layer-thicknesses. This remarkable feature helps the more accurate and consequent log interpretation, because there is a need to determine the textural parameters (e.g. cementation factor, tortuosity coefficient, saturation exponent) not only from literature or laboratory measurements but another objective source. It seems to me that it can be achieved by interval inversion in an advanced way that provides us layer-wise information about these parameters as a continuous log.

Since the interval inversion method uses an arbitrary set of basis function while series expansion it can be improved by employing orthogonal functions like trigonometric and Walsh functions or Chebishev and Legendre polynomials. Extending the petrophysical model, response functions of new well-logging tools can also be built up into the system. All taken altogether I can imagine this method to use for the joint inversion of well-logging data collected from several boreholes. For instance this multi-well application can serve the automatic solution of layer-correlation problems.

## IV. LIST OF RELATED PUBLICATIONS AND PRESENTATIONS

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